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TOPIC PLAN		
Partner organization	"Goce Delcev "- University Shtip, North Macadonia	
Topic	Differentiation	
Lesson title	Application in Business: Maximizing Revenue	
Learning objectives	The student will: • use first derivative to find extreme value • use second derivative to find maximum or minimal value • find the total revenue • Find the total profit	**Strategies/Activities** ☐Graphic Organizer ☐Think/Pair/Share ☐Modeling ☑Collaborative learning ☑Discussion questions ☑Project based learning ☑Problem based learning **Assessment for learning** ☑Observations ☑Conversations ☑Work sample ☐Conference ☐Check list ☐Diagnostics **Assessment as learning** ☐Self-assessment ☑Peer-assessment ☑Presentation ☐Graphic Organizer ☑Homework
Aim of the lecture / Description of the practical problem	A manufacturer of T-shirt determinates that in order to sell x units of a new T-shirt, the price per unit, in euros, must be $p(x)=\frac{16}{\sqrt{x}}$. The manufacturer also determines that the cost of producing x units is given by the function: $C(x)=2000+1.5x$ The manufacturer wants to find the price per unit that gives the maximum profit. For this problem, we need to find the price for which the manufacturer will have maximum profit.	
Previous knowledge assumed:	The student needs to know: • to calculate first and second derivates • to know differentiation Techniques: The Power and Sum–Difference Rules • to know differentiation Techniques: The Product and Quotient Rules • function for calculating profit • function for calculating revenue	

Introduction / Theoretical basics	The total cost $C(x) = \text{Fixed cost} + \text{Variable cost}$ where the fixed costs are indented of output and there are costs of fixed factors. The variable costs vary with the level of output and there are the costs of variable factors. The total revenue $R(x) = (\text{Number of units}) \cdot (\text{Price per units})$ The total profit $P(x) = \text{Total revenue -Total cost}$ To find the maximum value of the profit, we first need to find first derivate of $P(x)$, $P'(x)$. The critical value, we will find by solving the equation $P'(x) = 0.$ The value what is obtained is a critical value. We need to find second derivate to determinate whether this critical value is an absolute maximum. If a second derivates is negative $P''(x) < 0$, the profit will be maximized for this critical value. For a function $y = f(x)$ its derivative at x is the function f' defined by $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ provided that the limit exists. If $f'(x)$ exists, then we say that f is differentiable at x. The calculation of derivate of function is performed by help of the table for derivates of elementary	**Assessment of learning** ☑Test ☑Quiz ☑Presentation ☑Project ☐Published work

functions and by rules for calculation of derivates of functions that are not elementary.

The derivates of some elementary function are:

$$f(x)=c, \quad f'(x)=0$$
$$f(x)=x^n, \quad f'(x)=nx^{n-1}$$
$$f(x)=\frac{1}{x}, \quad f'(x)=-\frac{1}{x^2}$$
$$f(x)=\sqrt{x}, \quad f'(x)=\frac{1}{2\sqrt{x}}$$
$$f(x)=e^x, \quad f'(x)=e^x$$
$$f(x)=\ln x, \quad f'(x)=\frac{1}{x}$$
$$f(x)=a^x, \quad f'(x)=ax^{n-1}$$

Let $f(x)$ and $g(x)$ are differentiable functions and *c* is a constant. Then each of the following equations holds.

$(cf(x))'=cf'(x)$ - constant multiple rule

$(f(x)+g(x))'=f'(x)+g'(x)$ - sum rule

$(f(x)-g(x))'=f'(x)-g'(x)$ - difference rule

$(f(x)\cdot g(x))'=f'(x)\cdot g(x)+f(x)\cdot g'(x)$ - product rule

$\left(\frac{f(x)}{g(x)}\right)'=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{(g(x))^2}$ for $g(x)\neq 0$ - quotient rule

	The derivates are used for finding extreme value of the function. The function f has a minimum value at $x=c$ if $f^{'}(c)=0$ and $f^{''}(c)>0$. The function f has a maximum value at $x=c$ if $f^{'}(c)=0$ and $f^{''}(c)<0$.	
Action	The price per unit $p(x)$ is given by the function $p(x)=\frac{16}{\sqrt{x}}$. The total cost of producing x units is given by the function $C(x)=20+1.5x$. The total revenue $R(x)=(\text{Number of units})\cdot(\text{Price per units})$ $=x\cdot p=x\frac{16}{\sqrt{x}}=16\sqrt{x}$ The profit function $P(x)=\text{Total revenue -Total cost}$ $=R(x)-C(x)$ $=16\sqrt{x}-(20+1.5x)$ $=16\sqrt{x}-20-1.5x$ $=16\sqrt{x}-1.5x-20.$ To find the maximum profit, we need to find the maximum value of $P(x)$. For this goal, we first find	

$P^{'}(x)$.

$$P^{'}(x)=\frac{16}{2\sqrt{x}}-1.5.$$

The critical values will find from solving the equation:

$$P^{'}(x)=\frac{16}{2\sqrt{x}}-1.5=0$$

$$\frac{16}{2\sqrt{x}}=1.5$$

$$3\sqrt{x}=16$$

$$\sqrt{x}=\frac{16}{3}=5.33$$

$$x=5.33^2=28,44\approx 28$$

Second, we can therefore try to use the second derivative to determine whether we have an absolute maximum.

$$P^{''}(x)=\left(\frac{8}{\sqrt{x}}-1.5\right)^{'}=-\frac{4}{\sqrt{x^3}}.$$

Because of $P^{''}(x)$, is negative, the profit will be maximized when 28 units are produced and sold.

The maximum profit will be

$$P(28)=16\sqrt{x}-1.5x-20=22,66$$

T-shirt manufacturer makes a maximum profit of 22,66 euros by producing and selling 28 T-shirt.

The price per unit needed to make the maximum profit is

	$p(x) = \frac{16}{\sqrt{28}} = 3.$	
Materials / equipment / digital tools / software	The materials for learning are given as a part of references of the end from this topic plan; Equipment: classroom, green board, chalk in different colors; Digital tools: laptop, projector, smart board; Software: Mathematica.	
Consolidation	The students through the above example should understand that derivatives can be used to solve many problems in real life. Also, that the first derivatives are used to find extreme values, and with the second derivatives it is found whether this value is minimum or maximum. Give the similar problem of the students, but instead of to find maximizing revenue to find minimizing inventory costs.	

Reflections and next steps	
Activities that worked	**Parts to be revisited**
After the class, the teacher according to his personal perceptions regarding the success of the class fills in this part.	Through the success of the homework done by the students, questions and discussion at the beginning of the next class, the teacher comes to the conclusion which parts of this class should be revised.

References

[1] M. L. Bittinger, D. J. Ellenbogen and S.A. Surgent (2012), “Calculus and its applications”, Addison-Wesley
[2] G. Strang “Calculus” , Wellelye-Cambridge Press
[3] S. Calaway D. Hoffman and D.Lippman (2014) “Applied Calculus”
[4] P.D. Lax, M. S.Terrell (2014) “Calculus with Applications”, Springer